2. GOL'DENVEIZER A.L., The construction of an approximate theory of the bending of a plate by the method of asymptotic integration of the equations of elasticity theory. Prikl. Mat. Mekh., 26, 4, 1962.
3. KUCHERENKO V.V. and POPOV V.A., The asymptotic form of the solutions of problems of elasticity theory in thin regions. Dokl. Akad. Nauk SSSR, 274, 1, 1984.
4. AKSENTYAN O.K. and SELEZNEVA T.N., Determination of the eigenfrequencies of oscillations of circular plates. Prikl. Mat. Mekh., 40, 1, 1976.
5. GRINCHENKO V.T. and MELESHKO V.V., Harmonic Oscillations and Waves in Elastic Bodies, Nauk. Dumka, Kiev, 1981.
6. GUSEIN-ZADE M.I., On a plane problem of elasticity theory for a semi-strip. Prikl. Mat. Mekh., 41, 1, 1977.
7. OLEINIK O.A. and IOSIF'YAN G.A., On the decay conditions and the limit behaviour at infinity of solutions of a system of equations of elasticity theory. Dokl. Akad. Nauk SSSR, 258, 3, 1981.
8. NAZAROV S.A., Introduction to Asymptotic Methods of Elasticity Theory, Izd. LGU, Leningrad, 1983.
9. GREGORY R.D. and WAN F.Y.M., Decaying states of plane strain in a semi-infinite strip and boundary conditions for plate theory. J. Elasticity 14, 1, 1984.
10. DERESIEWICZ H. and MINDLIN R.D., Axially symmetric flexural vibrations of a circular disc. J. Appl. Mech. 222, 1, 1955.
11. GREGORY R.D. and WAN F.Y.M., On plate theories and Saint-Venant's principle. Intern. J. Solids and Structures, 21, 10, 1985.

Translated by H.T.

PMM U.S.S.R., Vol.52, No.5, pp.659-664, 1988
Printed in Great Britain

0021-8928/88 $10.00+0.00

ANTIPLANE DYNAMICAL CONTACT PROBLEM FOR AN ELECTROELASTIC LAYER*

O.D. PRYAKHINA and O.M. TUKODOVA

The antiplane dynamic contact problem of the excitation of a semibounded electroelastic layer with a lower boundary sharply constricted by a single electrode as the simplest transformer of electroelastic waves is considered. The electrode is modelled by an absolutely rigid polar stamp. In the region of contact between the electrode and the medium, the electric potential and the amplitudes of the shear displacements are given, and outside this region the surface is free from stress and normal component of the magnetic induction is equal to zero.

One of the approaches to studying the propagation laws for electroelastic shear waves in a medium and on a surface, where this approach is based on the use of the method of fictitious absorption is proposed. A comparative analysis of the behaviour of the basic characteristics of the problem for the coupled and uncoupled problems is given, and the behaviour of the amplitude-frequency dependence on the electrode width and the oscillation frequency is studied.

1. Let the medium occupy the region $-\infty \leqslant x, z \leqslant \infty, 0 \leqslant y \leqslant h$. As an electroelastic material, we consider an *XY*-cut of piezoelectric crystals of the 6*mm* hexagonal crystal symmetry class and a piezoelectric ceramic polarized along the *z*-axis. This case corresponds to the excitation of a shear surface waves $w_0(x, y)e^{-i\omega t}$.

The propagation of electroelastic shear waves in the quasistatic approximation for the

**Prikl.Matem.Mekhan.*, 52, 5, 844-849, 1988

materials under consideration, is described by the following system of equations in dimensionless amplitude parameters (the factor $e^{-i\omega t}$ that is common to all the characteristic is omitted here):

$$\Delta w + e\Delta\varphi + \Omega^2 w = 0, \quad e\Delta w - \varepsilon\Delta\varphi = 0 \tag{1.1}$$
$$\Delta = \partial_1^2 + \partial_2^2, \quad w = w(x_1, x_2), \quad \varphi = \varphi(x_1, x_2)$$

Let the mechanical loading $\tau(x_1)$ and the electric induction $d(x_1)$ be given on the surface of the medium $(x_2 = 1)$:

$$x_2 = 1, \quad \partial_2 w + e\partial_2\varphi = \tau(x_1), \quad e\partial_2 w - \varepsilon\partial_2\varphi = d(x_1) \tag{1.2}$$

and on the lower face of the layer $(x_2 = 0)$ the amplitudes of the shear displacements and of the electric potential become zero

$$x_2 = 0, \quad w = \varphi = 0 \tag{1.3}$$
$$e = e_{15}l/c_{44}^E, \quad \varepsilon = \varepsilon_{11}^S l^2/c_{44}^E, \quad \Omega^2 = \rho\omega^2 h^2 c_{44}^E$$
$$x = hx_1, \quad y = hx_2, \quad \varphi_0 = hl\varphi, \quad w_0 = hw, \quad \tau_0 = \tau c_{44}^E,$$
$$d_0 = c_{44}^E d/l$$

$c_{44}^E, e_{15}, \varepsilon_{11}^S$ are respectively elastic, piezoelectric and dielectric constants, ω is the oscillation frequency, h is the thickness of the layer, ρ is the material density φ, d, w, τ are the amplitudes of, respectively, the electric potential and induction, the shear displacement and the stress ($\varphi_0, w_0, \tau_0, d_0$ are dimensional qantities), l is a normalizing factor that has the dimensions of an electric field, x, y, z is a Cartesian system of coordinates, and ∂_1, ∂_2 denote differentiation with respect to x_1 and x_2, respectively.

The solution of (1.1) with conditions (1.2) and (1.3) is constructed using the Fourier transform method and takes the form

$$\mathbf{U}(\alpha, x_2) = \mathbf{K}(\alpha, x_2, \Omega)\mathbf{Q}(\alpha) \tag{1.4}$$
$$\mathbf{U}(\alpha, x_2) = \int_{-\infty}^{\infty} \mathbf{u}(x_1, x_2)\, e^{i\alpha x_1}\, dx_1, \quad \mathbf{Q}(\alpha) = \int_{-\infty}^{\infty} \mathbf{q}(x_1)\, e^{i\alpha x_1}\, dx_1$$
$$\mathbf{u} = \{w, \varphi\}, \; \mathbf{q} = \{\tau, d\}, \; \mathbf{U} = \{W, \Phi\}, \; \mathbf{Q} = \{T, D\}$$
$$\mathbf{K} = \begin{Vmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{Vmatrix}, \quad K_{11} = \frac{1}{\sigma c^2}\frac{\operatorname{sh}\sigma x_2}{\operatorname{sh}\sigma} \tag{1.5}$$
$$K_{12} = K_{21} = \frac{e}{\varepsilon}K_{11}, \quad K_{22} = \frac{e^2}{\varepsilon^2}K_{11} - \frac{\operatorname{sh}\alpha x_2}{\alpha\varepsilon\operatorname{ch}\alpha}$$
$$\sigma^2 = \alpha^2 - k^2, \quad k^2 = \frac{\Omega^2}{c^2}, \quad c^2 = 1 + \varkappa^2 \tag{1.6}$$
$$\varkappa^2 = \frac{e^2}{\varepsilon}, \quad k_e^2 = \frac{\varkappa^2}{1 + \varkappa^2}$$

The quantity k_e^2 is the square of the electromechanical coupling shear coefficient.

Applying an inverse Fourier transformation to (1.4), we obtain an integral representation of the solution of the problem of the propagation of electroelastic shear waves in the layer

$$\mathbf{u}(x_1, x_2) = \int_{-\infty}^{\infty} \mathbf{k}(x_1 - \xi, x_2)\,\mathbf{q}(\xi)\, d\xi \tag{1.7}$$
$$\mathbf{k}(x_1, x_2) = \frac{1}{2\pi}\int_\sigma \mathbf{K}(\alpha, x_2)\, e^{-i\alpha x_1}\, d\alpha$$

The contour σ is chosen in accordance with the radiation conditions in /1/.

2. Representation (1.7) gives an integral equation for the initial contact problem of the excitation of waves in an unbounded electroelastic layer by a single electrode of width $2b$, when on the surface of the medium

$$x_2 = 1, \quad \mathbf{u}(x_1, 1) = \mathbf{u}(x_1), \quad |x_1| \leqslant a \tag{2.1}$$
$$\mathbf{q}(x_1, 1) = 0, \quad |x_1| > a; \quad a = b/h$$
$$x_2 = 0, \quad \mathbf{u}(x_1, 0) = 0, \quad |x_1| < \infty$$

Thus, we have an integral representation of the first kind with respect to the unknown vector $\mathbf{q}$

$$\int_{-a}^{a} \mathbf{k}(x_1 - \xi)\,\mathbf{q}(\xi)\, d\xi = \mathbf{u}(x_1), \quad |x_1| \leqslant a \tag{2.2}$$

$$\mathbf{k}(x_1) = \frac{1}{2\pi}\int_\sigma \mathbf{K}(\alpha)\, e^{-i\alpha x_1} d\alpha, \quad \mathbf{K}(\alpha) = \mathbf{K}(\alpha, 1)$$

The elements of the matrix $\mathbf{K}(\alpha)$ are regular everywhere on the real axis, with the exception of the following poles which are the same for all functions: $u = \pm p_k$ $(k = 1, 2, \ldots, n)$, which are determined from (1.5) with $x_2 = 1$.

As $|\alpha| \to \infty$, we have the following asymptotic representation:

$$K_{ij}(\alpha) = c^{-2} |\alpha|^{-1} [a_{ij} + o(u^{-2})], \quad i, j = 1, 2$$

For piezoelectric materials of class 6*mm*, the coefficients $a_{11} = 1$, $a_{12} = a_{21} = e\varepsilon^{-1}$, $a_{22} = -\varepsilon^{-1}$.

With these properties for the kernel, system (2.2) is uniquely soluble in $L_\alpha(-a, a)$, $\alpha > 1$. The uniqueness criteria are proved by analogy with /1/. Without loss of generality, we will assume that

$$w(x_1) = A_1 e^{-i\eta x_1}, \quad \varphi(x_1) = A_2 e^{-i\eta x_1}, \quad \operatorname{Im}\eta = 0$$

We can construct the solution of (2.2) using the solution of the static and dynamic problems of the antiplane shear of an elastic layer. Both these problems reduce to solving an integral equation

$$\int_{-a}^{a} s(x_1 - \xi)\, t(\xi)\, d\xi = e^{-i\eta x_1}, \quad |x_1| \leqslant a \tag{2.3}$$

$$s(x_1) = \frac{1}{2\pi}\int_\sigma S(\alpha)\, e^{-i\alpha x_1} d\alpha$$

In the dynamics

$$S(\alpha) = \operatorname{th}\sqrt{\alpha^2 - k^2}/\sqrt{\alpha^2 - k^2}, \quad k^2 = \Omega^2 \tag{2.4}$$

In the statics $S(\alpha) = \operatorname{th}\alpha/\alpha$ $(\Omega = 0)$.

Let $t(x_1)$ be a solution of (2.3) with kernel (2.4), in which $k^2 = \Omega^2/(1 + \varkappa^2)$, while $t_0(x_1)$ corresponds to the solution for zero frequency Ω. Then the solution of (2.2) will be determined by the relationships

$$d(x_1) = (eA_1 - \varepsilon A_2) t_0(x_1) \tag{2.5}$$

$$\tau(x_1) = A_1 c^2 t(x_1) - e\varepsilon^{-1} d(x_1) \tag{2.6}$$

In statics, the contour σ coincides with the real axis.

The solution $t(x_1)$ of (2.3), (2.4) is constructed in /2/ by the method of fictitious absorption, which is also used in this paper. Another form of solution is given in /3/. The solution $t_0(x_1)$ is constructed in closed form in /4/.

It is obvious that the amplitude of the electric displacement (induction) (2.5), unlike the amplitude of the shear stresses (2.6), does not depend on the frequency Ω.

In the formulae given in /2-4/, the required functions $\tau(x_1)$ and $d(x_1)$ have singularities on the boundary $\sqrt{x_1 \pm a}$.

3. In calculating structures using the coupling of electric and mechanical fields, in particular, structures on surface acoustic waves, we often neglect the contribution of elastic waves to the electric induction because of the smallness of the electromechanical coupling coefficient. In this case, the solution of the problem has the form

$$d(x_1) = -\varepsilon A_2 t_0(x_1) \tag{3.1}$$

$$\tau(x_1) = A_1 t_*(x_1) + eA_2 t_0(x_1) \tag{3.2}$$

Here $t_*(x_1)$ is a solution of (2.3) with kernel (2.4), that is, a solution of the dynamical problem of the antiplane shear of an eleastic layer.

With the given null potential $(A_2 = 0)$ the shear stresses will not depend on the electric properties of the medium and the behaviour of $\tau(x_1)$ will be identical with the corresponding characteristic of the purely elastic problem, where $d(x_1) \equiv 0$. With $A_2 \neq 0$, the function $\tau(x_1)$ will be determined by the superposition of solutions of the static and dynamic problems of elasticity theory. The amplitude of the electric induction for the uncoupled problem will, as before, remain a real quantity and will not depend on the frequency.

In Figs.1 and 2 we show the dependence of $\operatorname{Re}\tau$ and d on x_1 for coupled (the solid lines) and uncoupled (the dashed lines) electromechanical problems with $A_1 = A_2 = 1$ $(\eta = 0)$, $\Omega = 4$, $a = 5$. Curves *1, 2, 3* correspond to CdS $(k_e = 0.19)$, ZnO $(k_e = 0.32)$ and TsTS-19 $(k_e = 0.58)$. In Fig.1, the behaviour of $\operatorname{Re}\tau(x_1)$ is given, for comparison, by the dashed line for the purely elastic problem, which corresponds to (3.2) with $A_1 = 1$, $A_2 = 0$ $(\eta = 0)$. The divergence between the distribution of contact stresses and the electric induction in the contact zone for materials with a different electromechanical coupling coefficient increases as k, increases. For piezoelectric crystals that have a small electromechanical coupling coefficient we can, as for Cd S, neglect the contribution of the elastic waves to the electric induction, which considerably simplifies the construction of solutions to such problems.

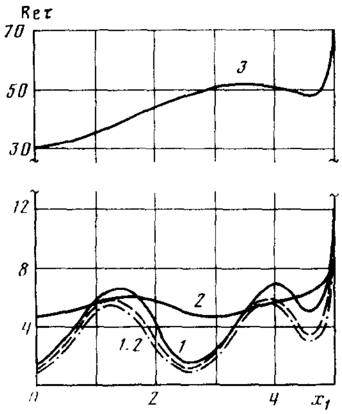

Fig.1

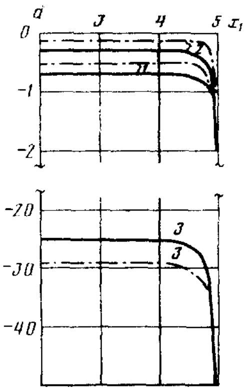

Fig.2

4. On the real axis $K_{ij}(\alpha)$ have the same poles $\alpha = \alpha(\Omega)$ for all the functions, and these poles correspond to the value of the wave number of the surface wave propagating on the surface of the piezoelectric. These poles also determine the dispersion curves of the problem

$$\alpha = [\Omega^2 (1 + \varkappa^2)^{-1} - \pi^2 (1/2 + n)^2]^{1/2}, \quad n = 0, 1, \ldots$$

which at any frequency enable us to find the number and phase velocities of the surface waves that arise

$$V = \sqrt{c_{44}/\rho}\,\Omega/\alpha = V_t \Omega/\alpha$$

(V_t is the velocity of transverse volume waves ignoring the piezoelectric effect).

The locking frequencies (the frequencies at which standing waves are formed with $\alpha = 0$ /5/) are

$$\Omega = \sqrt{1 + \varkappa^2}\,(\pi/2 + \pi n), \quad n = 0, 1, \ldots$$

It is obvious that the parameter $\varkappa^2$ introduces a correction to the phase velocity of propagation of the shear waves, which leads to an increase in this phase velocity. We note that the dispersion curves for the uncoupled problem and the corresponding elastic problem coincide.

5. Knowing the distribution of shear stresses and of the electric induction in the contact region we can determine the electroelastic wave field that arises in the medium and on the surface. After substituting (1.5), (2.5) and (2.6) into (1.7), we have

$$w(x_1, x_2) = \frac{A_1}{2\pi} \int_\sigma \frac{\operatorname{sh} \sigma x_2}{\sigma \operatorname{ch} \sigma} T(\alpha) e^{-i\alpha x_1} d\alpha \tag{5.1}$$

$$\varphi(x_1, x_2) = \frac{e}{\varepsilon} w(x_1, x_2) - \frac{eA_1 - \varepsilon A_2}{2\pi\varepsilon} \int_{-\infty}^{\infty} \frac{\operatorname{sh} \alpha x_2}{\alpha \operatorname{ch} \alpha} T_0(\alpha) e^{-i\alpha x_1} d\alpha$$

The shear wave is piezoactive and, together with the potential of the electric field, it is described by an oscillating function.

In the uncoupled problem

$$w(x_1, x_2) = \frac{A_1}{2\pi} \int_\sigma \frac{\operatorname{sh} \sigma_0 x_2}{\sigma_0 \operatorname{ch} \sigma_0} T_*(\alpha) e^{-i\alpha x_1} d\alpha, \quad \sigma_0^2 = \alpha^2 - \Omega^2 \tag{5.2}$$

$$\varphi(x_1, x_2) = \frac{A_2}{2\pi} \int_{-\infty}^{\infty} \frac{\operatorname{sh} \alpha x_2}{\alpha \operatorname{ch} \alpha} T_0(\alpha) e^{-i\alpha x_1} d\alpha$$

It is obvious that in the case of the uncoupled problem the potential is a decreasing function of x_1 (as in the static elastic case), and the shear wave ceases to be piezoactive.

We note that $T(\alpha)$, $T_0(\alpha)$, $T_*(\alpha)$ are the Fourier transforms of the functions $t(x_1)$, $t_0(x_1)$, $t_*(x_1)$ respectively.

The integrals in (5.1) and (5.2) are calculated by integrating over the rectangular contour σ by analogy with /6/. In the far zone, it is more convenient to use the asymptotic formulae.

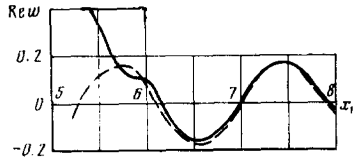

Fig.3

Fig.3 shows the dependence of $\mathrm{Re}\, w$ on x_1 for CdS ($k_e = 0.19$, $\Omega = 4$, $a = 5$, $A_1 = A_2 = 1$). The solid curve is obtained by integration using (5.1) along the contour of σ, and the dashed line is obtained using the theory of residues (the integrand in (5.1) has a single pole for the given frequency $\alpha = 3.6$). It is obvious that with $x_1 > 6.5$ it is more convenient to calculate the wave field using asymptotic formulae. We note that in the region of contact between the electrode and the medium the values of w, φ are close to unity. The maximum error of about 10% occurs at the edge of the electrode.

6. We now consider the purely "electrical" problem, where the following boundary conditions are specified on the surface of the medium in place of conditions (2.1):

$$x_2 = 1, \quad \varphi(x_1, 1) = \varphi(x_1), \quad |x_1| \leqslant a \qquad (6.1)$$
$$d(x_1) = 0, \quad |x_1| > a; \quad \tau(x_1) = 0, \quad -\infty < x_1 < \infty$$

In this case we have an integral equation for the unknown function $d(x_1)$:

$$\int_{-a}^{a} k(x_1 - \xi)\, d(\xi)\, d\xi = \varphi(x_1), \quad |x_1| \leqslant a \qquad (6.2)$$
$$k(x_1) = \frac{1}{2\pi} \int_\sigma K_{22}(\alpha)\, e^{-i\alpha x_1}\, d\alpha$$
$$K_{22}(\alpha) = \frac{1}{\varepsilon}\left(\frac{\varkappa^2}{1+\varkappa^2}\frac{\operatorname{th}\sigma}{\sigma} - \frac{\operatorname{th}\alpha}{\alpha}\right)$$

The solution of (6.2) for $\varphi(x_1) = Ae^{-i\eta x_1}$ also has the form given in /2/, where the function $K_{22}(\alpha)$ is written in the form

$$K_{22}(\alpha) = -\frac{1}{\sqrt{\alpha^2 + B^2 \varepsilon c^2}} \prod_{k=1}^{n} \frac{\alpha^2 - z_k^2}{\alpha^2 - p_k^2}$$

where z_k, p_k $(k = 1, 2, \ldots, n)$ are respectively real and complex zeros, and the poles of $K_{22}(\alpha)$ located above the contour σ.

The electroelastic wave field will be described by (1.7), which, taking account of (6.1) after determining $d(x_1)$, we can write in the form

$$w(x_1, x_2) = \frac{eA}{2\varepsilon c^2 \pi} \int_\sigma \frac{\operatorname{sh}\sigma x_2}{\sigma \operatorname{ch}\sigma} D(\alpha)\, e^{-i\alpha x_1}\, d\alpha \qquad (6.3)$$

$$\varphi(x_1, x_2) = \frac{A}{2\pi\varepsilon}\left\{ew(x_1, x_2) - \int_{-\infty}^{\infty} \frac{\operatorname{sh}\alpha x_2}{\alpha \operatorname{ch}\alpha} D(\alpha)\, e^{-i\alpha x_1}\, d\alpha\right. \qquad (6.4)$$

Fig.4 shows the dependence of $\mathrm{Re}\, d$ on x_1 for various widths of the electrode, measured in terms of the wavelength λ ($a = \lambda$, $\lambda/2$, $\lambda/4$) with $\Omega = 4$ for TsTS-19 (curves *1, 2, 3*). We note that in this case $d(x_1)$ is an oscillating function and depends on the function Ω, unlike the corresponding characteristic of the electromechanical problem (Fig.2).

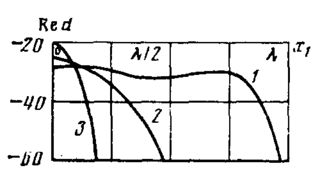

Fig.4

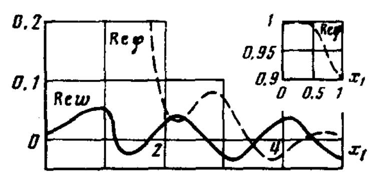

Fig.5

Fig.5 shows the behaviour of the amplitude functions $w(x_1, 1)$ and $\varphi(x_1, 1)$ with $x_2 = 1$, $\Omega = 4$, $a = 1$ ($A = 1$, $\eta = 0$) for ZnO. In the region of contact between the electrode and the medium, the quantity $\varphi(x_1, 1)$ is close to unity, that is, to the specified potential amplitude, and the quantity $w(x_1, 1)$ is close to zero. The maximum error is on the edge of the electrode (of the order of 20% for $a = 1$) and decreases as the width $2a$ of the electrode increases (for $a = 5$ the error near the edge is 10%).

The author thanks I.I. Vorovich for discussing this paper and for useful remarks.

REFERENCES

1. VOROVICH I.I. and BABESHKO V.A., Dynamic mixed problems of the Theory of Elasticity for Non-classical Regions, Nauka, Moscow, 1979.

2. BABESHKO V.A. and PRYAKHINA O.D., The fictitious absorption method in plane dynamic problems. Prikl. Mat. Mekh. 44, 3, 1980.
3. ALEKSANDROV V.M. and KOVALENKO E.V., Problems of the Mechanis of Continuous Media with Mixed Boundary Conditions, Nauka, Moscow, 1986.
4. ALEKSANDROV V.M. and CHEBAKOV M.I., On the method of homogeneous solutions in mixed problems of the theory of elasticity for a truncated wedge and an annular sector. Prikl. Mat. Mekh. 47, 5, 1983.
5. GRINCHENKO V.T. and MELESHKO V.V., Harmonic Oscillations and Waves in Elastic Bodies, Nauk. Dumka, Kiev, 1981.
6. GLUSHKOVA N.V., KALINCHUK V.V., SELEZNEV M.G. and others. Procedure and results of calculations of the wave field near a vibrational source located on the surface of a homogeneous semispace, In: Problems of the Excitation of Seismic waves by a Source of Vibrations Inst. Geol. i Geofiz. SO Akad. Nauk SSSR, Novosibirsk, 1976.

Translated by H.T.

PMM U.S.S.R.,Vol.52,No.5,pp.664-669,1988
Printed in Great Britain
0021-8928/88 $10.00+0.00

ASYMPTOTIC PROPERTIES OF THE APPROXIMATE SOLUTION OF A CLASS OF DUAL INTEGRAL EQUATIONS*

S.M. AIZIKOVICH and I.S. TRUBCHIK

An investigation is presented of dual integral equations generated by various plane contact problems: a strip resting without friction on a rigid base (Problem 1), a strip clamped along the base (Problem 2), a wedge with a clamped face (Problem 3), and also be axisymmetric problems relating to the action of a ring-shaped stamp on a half-space (Problem 4), and the interaction of an elastic bandage with an elastic cylinder /1/ (Problem 5). The strip, wedge, half-space and cylinder may be uniform, laminar of continuously inhomogeneous. Analogous equations in terms of Laplace transforms are obtained in problems of coupled thermo-elasticity and consolidation theory of water-saturated media for the bodies listed here /2/.

The method described in /3/ is generalized to construct solutions of the above problems. Well-posedness and solvability classes are established for the equations, proving that the approximate method proposed here is asymptotic in both directions with respect to a characteristic geometric parameter $\lambda = H/a$ (H is the thickness of the strip and a is half the thickness of the stamp) in Problems 1 and 2, or $\lambda = 2/\ln(b/a)$ (a and b are the distances from the nearest and farthest points at which the stamp touches the boundary of the wedge to its vertex) in Problem 3, $\lambda = 2/\ln(b/a)$ (a is the inner radius of the stamp and b its outer radius) in Problem 4, $\lambda = R/a$ (R is the radius of the cylinder and a half the thickness of the bandage) in Problem 5. In problems of coupled thermo-elasticity and consolidation theory λ also involves the parameter p of the Laplace transform with respect to the time coordinate /2/. The method is illustrated in relation to a contact problem for a strip continuously inhomogeneous in depth.

1. Statement of the problem.

Consider the dual integral equation

$$\int_{-\infty}^{\infty} \Phi(\alpha) \frac{\operatorname{th}(A\lambda\alpha)}{\alpha} L(\lambda\alpha) e^{-i\alpha x} d\alpha = 2\pi g(x), \quad |x| \leqslant 1 \tag{1.1}$$

**Prikl.Matem.Mekhan.*,52,5, 850-856,1988